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Methods of Modeling of The Poliform Cone Surfaces

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SUMMARY

In this paper, are proposed two methods of poliform cone surfaces geometrical modeling, based on algorithms that apply on kinematic principles of generation, treated graphically and analytically.
***Keywords**: poliform cone surfaces, modelling*

1. Introduction

Poliform surfaces have different forms, depending on their destination, existing standardisations of their form and dimension.

Mathematical methods for the study of poliform surfaces are analytical, based on general theorems of surface winding.

In the same time, the developpment of the graphical methods in AUTOLISP programming medium, allow the insertion of the solid modeling for the study of this kind of problems as well.

In this paper, are given solutions for poliform cone surfaces modeling, generated based on a kinematic that uses as generator parts edged by primary surfaces of revolution.

2. Method of the solid modeling

A. Kinematics of the generation process

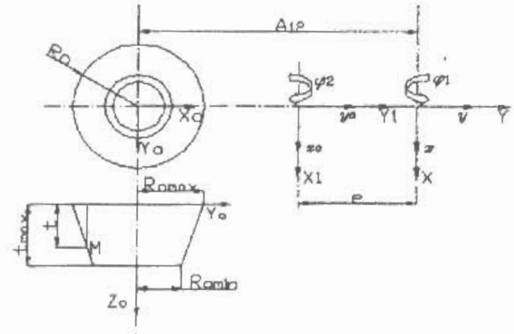


Fig. 1. Reference systems

In figure 1, is presented the kinematic sheet of generation of poliform cone surfaces through a method of planetary grinding.

The semi-product processed through grinding, in the assembly of the rotary motion I and II, of parallel axis and angular speed corelated one with another, by the cone abrazive wheel having its axis placed at the distance A_{12} towards the rotatory axis of the planetary system crank.

The excentricity of the planetary motion is noted "e".

In order to transcript the matrix form of motions assembly are defined the reference systems, figure 2:

xyz is the fixed reference system, with z axis solidary to the rotatory axis of the planetary mechanism crank;

XYZ is a mobile system solidary to $X_0Y_0Z_0$ in rotation of angle φ_1

$x_0y_0z_0$ is a mobile system solidary to XYZ, in its rotatory motion

$X_1Y_1Z_1$ is a mobile system solidary to the generated semi-product

$X_0Y_0Z_0$ is the fixed reference system solidary to the solid of the abrazive part.

Are defined the motions:
rotation of the system XYZ,

$$x = \omega_3^T(\varphi_1) \cdot X; \qquad (1)$$

rotation of the system $X_1Y_1Z_1$:

$$x_0 = \omega_3^T(-\varphi_2) \cdot X_1; \qquad (2)$$

link between the systems $x_0y_0z_0$ and XYZ
$x_0 = x - a$

$$\text{where} \quad a = \begin{Bmatrix} 0 \\ e \\ 0 \end{Bmatrix} \qquad (3)$$

addiction between the fixed systems xyz and $X_0Y_0Z_0$

$$\text{where} \quad b = \begin{Bmatrix} 0 \\ A_{12} \\ 0 \end{Bmatrix}. \qquad (4)$$

Results the relative motion of the system $X_0Y_0Z_0$ in comparison with the system $X_1Y_1Z_1$ of form

$$X_1 = \omega_3(-\varphi_2)\cdot[\omega_3(\varphi_1)\cdot(X_0 + b) - a]. \quad (5)$$

B. "Primitive" solid of the abrazive part

The primitive, representing the abrazive part, can be realized through the rotation of the generatrix "g" around Z_0 axis. Generatrix "g" is of equations:

$$g\begin{cases} X_0 = 0 \\ Y_0 = R_{0\,max} - tg\chi \\ Z_0 = t \end{cases} \qquad (6)$$

where: t- variable,
and through the rotation of the generatrix "g" around the axis Z_0, results the relation

$$\begin{Bmatrix} X_0 \\ Y_0 \\ Z_0 \end{Bmatrix} = \begin{Bmatrix} \cos\varphi_1 & -\sin\varphi_1 & 0 \\ \sin\varphi_1 & \cos\varphi_1 & 0 \\ 0 & 0 & 1 \end{Bmatrix} \cdot \begin{Bmatrix} 0 \\ R_{0\,max} - tg\chi \\ t \end{Bmatrix} \qquad (7)$$

C. Solid modeling of the poliform surface

If it is considered a point M of the primitive "S' of coordinates

$$M = \begin{Bmatrix} 0 \\ R_{0\,max} \\ 0 \end{Bmatrix} \qquad (8)$$

than the transforming (5) represents the relative trajectory of the point M and, in same time, of the primitive solid towars the reference system of the generated semi-product

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} \cos\varphi_2 & -\sin\varphi_2 & 0 \\ \sin\varphi_2 & \cos\varphi_2 & 0 \\ 0 & 0 & 1 \end{Bmatrix} \cdot \left[\begin{Bmatrix} \cos\varphi_1 & \sin\varphi_1 & 0 \\ -\sin\varphi_1 & \cos\varphi_1 & 0 \\ 0 & 0 & 1 \end{Bmatrix} \cdot \begin{Bmatrix} 0 \\ R_{0\,max} + A_{12} \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ e \\ 0 \end{Bmatrix} \right] \qquad (9)$$

In this relative motion, the solid determined through the relation (7) is subtracted from the initial cylindrical part of the semi-product, modeling in this way its effective form. In order to do this, it is used line (command "subtract") in source code AUTOLISP with the variation of the parameter φ_1 between [0.6 π] and the transmission ratio i=4/3, and φ_2 =i φ_1

In figure 2, it is presented the primitive model and its successive positions towards the semi-product.

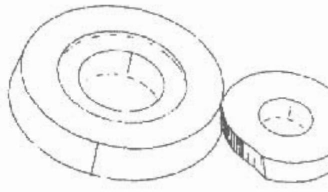


Fig. 2. Primitive "tool" in comparison with the semi-product

In figure 3, it is presented the solid model of the three sided cone poliform surface, for the case when the transmission ratio i =3/4.

Given:
A_{12}=200 mm;
R_{max}=120 mm;
t_{max}=50 mm
e=100 mm

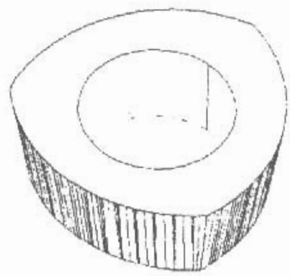


Fig. 3. Solid metod

3. Analytical method

As a variant for the previous generation model it is developped an analytical form of modeling the same type of surface.

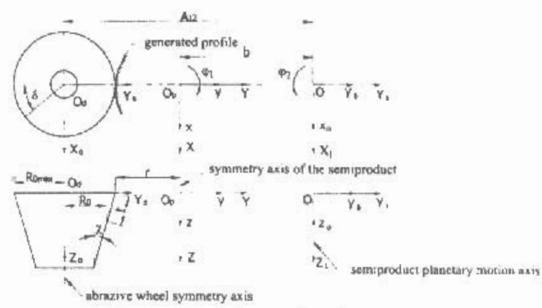


Fig. 4. Generation sheet

In figure 4, are presented the reference systems and the motions, in the planetary generation process of a noncylindrical poliform shaft:

$x_0y_0z_0$ fixed reference system, with z_0 axis of rotation in semi-product's rotatory motion;

xyz helping reference system, solidary to the $X_1Y_1Z_1$ mobile reference system, in rotatory motion around the axis z_0 of the system $x_0y_0z_0$ in rotatory motion;

$X_0Y_0Z_0$ fixed reference system, solidary to the generator abrazive part

XYZ mobile reference system, solidary to the semi-product, in rotatory motion around the system xyz.

The cone abrazive part, realizes a rotatory motion around its own axis.

Parametric equations of the peripherical surface that generates the abrazive cone part:

$$S:\begin{cases} X_0 = \left[R_{0\,max} - t\cdot sin\,\chi\right]\cdot cos\,\delta; \\ Y_0 = \left[R_{0\,max} - t\cdot sin\,\chi\right]\cdot sin\,\delta; \\ Z_0 = t\cdot cos\,\chi, \end{cases} \qquad (10)$$

where:

t, δ are variable parameters, $t\in\left[0;\dfrac{h}{cos\,\chi}\right]$;

h- semi-product's thickness;

χ - angle at the top of the generator cone;

R_{0max} maximal radius of the abrazive wheel.

The rotatory motion of the system $X_1Y_1Z_1$, of angle φ_2, towards the axis z_0 is given by the equation

$$x_0 = \omega_3^T(\varphi_2)\cdot X_1 \qquad (11)$$

where φ_2 is a variable parameter.

As well,

$$x = \omega_3^T(-\varphi_1)\cdot X. \qquad (12)$$

represents the rotatory motion of the semi-product's system processed in comparison with Z axis, of angular parameter φ_1.

Are defined the following transformings of coordinates that establish the relative positions of the reference systems:

$$X_1 = x - a, \qquad (13)$$

where: $a=\begin{Bmatrix}0\\ b\\ 0\end{Bmatrix}$;

b- radius of the planetary motion of generation of the poliform surface;

$$X_0 = x_0 - B; \qquad (14)$$

$$a=\begin{Bmatrix}0\\ -A_{12}\\ 0\end{Bmatrix};$$

A_{12}- distance from the planetary motion axis to the abrazive wheel axis.

Relations (13) and (14), and (11), (12), allow the determination of the relative motion of the system $X_0Y_0Z_0$ of the abrazive part system towards the reference system XYZ of the semi-product:

$$X = \omega_3(-\varphi_1)\cdot\left[\omega_3(\varphi_2)\cdot(X_0 + B) + a\right]. \qquad (15)$$

Between the two parameters φ_1 and φ_2 is established a relation of form

$\varphi_1 = i\cdot\varphi_2$, where i – the transmission ratio: $i=\dfrac{3}{4}$ for shafts with 3 sides and $i=\dfrac{4}{3}$ for shafts with 4 sides.

The family of cone surfaces of the abrazive part it is determined with the relations

$$\begin{Bmatrix}X\\ Y\\ Z\end{Bmatrix} = \begin{Bmatrix}cos\,\varphi_1 & -sin\,\varphi_1 & 0\\ sin\,\varphi_1 & cos\,\varphi_1 & 0\\ 0 & 0 & 1\end{Bmatrix}\cdot\left[\begin{Bmatrix}cos\,\varphi_2 & sin\,\varphi_2 & 0\\ -sin\,\varphi_2 & cos\,\varphi_2 & 0\\ 0 & 0 & 1\end{Bmatrix}\left(\begin{Bmatrix}\left[R_{0max} - t\cdot sin\,\chi\right]\cdot cos\,\delta\\ \left[R_{0max} - t\cdot sin\,\chi\right]\cdot sin\,\delta\\ t\cdot cos\,\chi\end{Bmatrix} + \begin{Bmatrix}0\\ -A_{12}\\ 0\end{Bmatrix}\right) + \begin{Bmatrix}0\\ b\\ 0\end{Bmatrix}\right]. \qquad (16)$$

The envelope of this surfaces family is the peripherical surface of the generated poliform shaft.

The problem of the envelope can be solved trough one of these methods: Gohman method, method of the "minimum distance", "trajectories" method.

Through successive transformings it will arrive to the next system of equations, that defines the trajectory of a point from the space of the system $X_0Y_0Z_0$ Towards the reference system XYZ:

$$T_S:\begin{cases} X = \left[R_{0max} - t\cdot sin\,\chi\right]\cdot cos[(i-1)\cdot\varphi_1 - \delta] - \\ \quad - A_{12}\cdot sin[(i-1)\cdot\varphi_1] - b\cdot sin\,\varphi_1; \\ Y = \left[R_{0max} - t\cdot sin\,\chi\right]\cdot sin[(i-1)\cdot\varphi_1 - \delta] \\ \quad - A_{12}\cdot cos[(i-1)\cdot\varphi_1] + b\cdot sin\,\varphi_1; \\ Z = t\cdot cos\,\chi. \end{cases} \qquad (16')$$

The winding condition for the cross-sections,

$$Z = t\cdot cos\,\chi \qquad (17)$$

of form

$$\frac{X'_\theta}{X'_{\varphi_1}} = \frac{Y'_\theta}{Y'_{\varphi_1}}, \qquad (18)$$

where X'_θ, X'_{φ_1}, Y'_θ, Y'_{φ_1} partial derivative of the equations (16') in comparison with the variables θ and φ_1, can be brought to the expression

$$\delta = arctg\left[\frac{A_{12}\cdot(1-i) - b\cdot cos(i\cdot\varphi_1)}{b\cdot sin(i\cdot\varphi_1)}\right]. \qquad (19)$$

For any value of t, the assembly of equations (16') and (19) represent a family of plane sections of the generated poliform shaft- poliform cone shaft.

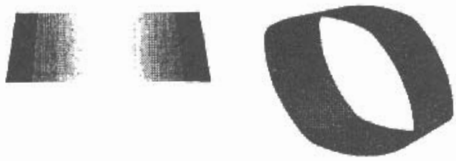
In figures 6 and 7, are presented the forms of the cone poliform surfaces modelled in accordance with the presented algorithm.

Three solid poliform profile:

r_{semif}=60 mm; r=45 mm; χ=10°; b=20 mm;

R_{0max}=200 mm; h=30 mm; $i=\frac{3}{4}$; n_s=5 secţiuni; n=50 puncte/ secţiune; A_{12}=265 mm; t_{max}=30,46 mm; φ_{1max}=60°; $pas_{\varphi 1}$=1,12°.

r_{semif}=50 mm; r=40 mm; χ=8°; b=15 mm; R_{0max}=160 mm; h=35 mm; n_s=5 secţiuni; n=50 puncte/ secţiune; $i=\frac{4}{3}$; A_{12}=215 mm; t_{max}=35,34 mm; φ_{1max}=60°; $pas_{\varphi 1}$=1,12°.



4. Conclusions

The proposed kinematics and the realized soft products allow the bidimensional and 3D modeling of the cross-sections form and the assembly form of the cone poliform surface.

The original softs, dedicated to this purpose, are flexible enough to allow the modeling of any type of solid or surface in this category.

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Metode de Modelare a Suprafeţelor Poliforme Conice

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Rezumat

În lucrare, se prezintă un model geometric, în baza unei scheme de generare utilizând suprafaţa conică exterioară a unui corp abraziv, al unei suprafeţe poliforme conice.

Sunt prezentate, de asemenea, două soluţii de modelare: o soluţie analitică, dezvoltată în baza teoriei înfăşurării suprafeţelor şi o soluţie utilizând principiile modelării solide.

La Méthode Du Modélage Des Surfaces Poliformes Conuques

Université "Dunărea de Jos" Galaţi

Résumé

Dans la papier,on présente un modéle géometriqe, basé sur une schema de génération en utilisantla surface conique extereiure d'un corps abrasif, d'une surface poliforme conique.

On a présenté, aussi, deux modalités de modélage: un modalité analitique,basée sur la théorie d'enveloppementdes surfaces et l'antre en utilisant les principes du modelage solide.